

Soluții

1. a) $A^2 = \begin{pmatrix} a^2 + bc & ab + bd \\ ac + cd & bc + d^2 \end{pmatrix}.$

b) $(a+d)A = \begin{pmatrix} a^2 + ad & ab + bd \\ ac + cd & ad + d^2 \end{pmatrix}; (a+d)A - (ad-bc)I_2 = \begin{pmatrix} a^2 + bc & ab + bd \\ ac + cd & bc + d^2 \end{pmatrix}.$

c) $A^2M = (a+d)AM - (ad-bc)M; MA^2 = (a+d)MA - (ad-bc)M; a+d \neq 0 \Rightarrow AM = MA.$

2. a) $f = X^3 - 2X^2 + X; f = X^3 - 2X^2 + X; f = X(X-1)^2; x_1 = 0, x_2 = x_3 = 1.$

b) $x_1 + x_2 + x_3 = 2, x_1x_2 + x_1x_3 + x_2x_3 = a; x_1^2 + x_2^2 + x_3^2 = 2 \Leftrightarrow 4 - 2a = 2 \Leftrightarrow a = 1.$

c) $x_1^2x_2^2x_3^2 = b^2 = -b \Rightarrow b = 0 \text{ sau } b = -1; x_1^2 + x_2^2 + x_3^2 = x_1 + x_2 + x_3 \Leftrightarrow a = 1; a = 1 \text{ și } b = 0.$