

Soluții

1. a) $A^2 = \begin{pmatrix} 4 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 2 & 1 \end{pmatrix}.$

b) $A^3 = \begin{pmatrix} 8 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 3 & 1 \end{pmatrix}; 4A^2 - 5A + 2I_3 = \begin{pmatrix} 8 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 3 & 1 \end{pmatrix}.$

c) $\det A = 2 \neq 0 \Rightarrow A^{-1} = \begin{pmatrix} \frac{1}{2} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 1 \end{pmatrix}, mA^2 + nA + pI_3 = \begin{pmatrix} 4m + 2n + p & 0 & 0 \\ 0 & m + n + p & 0 \\ 0 & 2m + n & m + n + p \end{pmatrix}.$

Identificând elementele obținem $m = \frac{1}{2}, n = -2, p = \frac{5}{2}.$

2. a) A doua ecuație $\Leftrightarrow \frac{x_1x_2 + x_2x_3 + x_3x_1}{x_1x_2x_3} = \frac{1}{2}; x_1x_2x_3 = -4.$

b) $s_1 = x_1 + x_2 + x_3, s_2 = x_1x_2 + x_1x_3 + x_2x_3, s_3 = x_1x_2x_3;$ ecuația $x^3 - s_1x^2 + s_2x - s_3 = 0$ ecuația cerută
 $x^3 - 2x^2 - 2x + 4 = 0$, deci $a = -2, b = -2, c = 4.$

c) $f = (x-2)(x^2-2) = (x-2)(x-\sqrt{2})(x+\sqrt{2}).$