

Soluții

1. a) $X^2 = \begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{pmatrix}$

b) $\det(X) = 1, X' = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{pmatrix}, X^{-1} = \begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix}.$

c) $X^3 = \begin{pmatrix} 1 & 3 & 6 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{pmatrix}, 3X^2 + rX + I_3 = \begin{pmatrix} r+4 & 6+r & 9+r \\ 0 & r+4 & 6+r \\ 0 & 0 & r+4 \end{pmatrix}.$ Identificând elementele, $\forall a, b, c \in \mathbb{R},$

obținem $r = -3.$

2. a) $2008 \circ (-2008) = 2^0 = 1.$

b) $x \circ x^2 = 64 \Leftrightarrow x + x^2 = 6 \Leftrightarrow x \in \{-3, 2\}.$

c) $(x \circ y) \circ z = 2^{x+y} \circ z = 2^{2^{x+y}+z}; 2^{2^{x+y}+z} = 2^{z+1} \Rightarrow 2^{x+y} + z = z + 1 \Rightarrow x + y = 0 \Rightarrow x = -y.$