Soluții

1. a) $M_1 + M_2 = \begin{pmatrix} 2 & 3 \\ 0 & 2 \end{pmatrix}; \det(M_1 + M_2) = 4$.

b) $M_a^2 = \begin{pmatrix} 1 & 2a \\ 0 & 1 \end{pmatrix}$.

c) $X = \begin{pmatrix} x & y \\ z & t \end{pmatrix}$, $M_a X = \begin{pmatrix} x+az & y+at \\ z & t \end{pmatrix}$, $XM_a = \begin{pmatrix} x & ax+y \\ z & az+t \end{pmatrix}$;

$x + az = x, y + at = ax + y, z = z, t = az + t, \forall a \in \mathbb{R}$; obținem $z = 0$; $t = x$, deci $X = \begin{pmatrix} x & y \\ 0 & x \end{pmatrix}$, pentru oricare $x, y \in \mathbb{R}$.

2. a) $x * 0 = x$.

b) $x*(y*z) = x*\sqrt[3]{y^3+z^3} = \sqrt[3]{x^3+y^3+z^3}$; $(x*y)*z = \sqrt[3]{x^3+y^3}*z = \sqrt[3]{x^3+y^3+z^3}$.

c) $x_1 = \sqrt[3]{2x_0^3} = \sqrt[3]{2}x_0$. Prin inducție: $x_n = \sqrt[3]{n+1} \cdot x_0$; $x_7 = \sqrt[3]{8}x_0 = 2x_0 \in \mathbb{Q}$, pentru $x_0 \in \mathbb{Q}$.

$x_2 = x_0 * x_1 = \sqrt[3]{x_0^3 + 2x_0^3} = x_0\sqrt[3]{3}$; $x_3 = x_0 * x_2 = \sqrt[3]{x_0^3 + 3x_0^3} = x_0\sqrt[3]{4}$, $x_0 \in \mathbb{Q}$ și $\sqrt[3]{4} \notin \mathbb{Q} \Rightarrow x_3 \notin \mathbb{Q}$.