Soluții

1. **a)** $A(1)\cdot A(-1)=A(-1)$.

b) $\left(A(x)\right)^2=\begin{pmatrix}5x^2+10x+1 & -2x^2-4x\\ 10x^2+20x & -4x^2-8x+1\end{pmatrix}=A\left((x+1)^2-1\right),\ \forall x\in\mathbb{R}$.

c) $\det A(1)=2\neq 0.$ Deci matricea $A(1)$ este inversabilă. $A(1)^{-1}=\frac{1}{2}\begin{pmatrix}-3 & 2\\ -10 & 6\end{pmatrix}$.

2. **a)** $0=0+0\sqrt{3}\notin G$ pentru că $0^2-3\cdot 0^2\neq 1$.

$1=1+0\sqrt{3},\quad 0,1\in\mathbb{Z}$ şi $1^2-3\cdot 0^2=1.$ Deci $1\in G$.

b) Fie $x,y\in G$

$x=a+b\sqrt{3},\quad a,b\in\mathbb{Z},\ a^2-3b^2=1,\ \ y=c+d\sqrt{3},\ \ c,d\in\mathbb{Z},\ c^2-3d^2=1$

$xy=ac+3bd+(ad+bc)\sqrt{3}.$ Avem $ac+3bd, ad+bc\in\mathbb{Z}$ si $(ac+3bd)^2-3(ad+bc)^2=1$.

De unde rezultă concluzia.

c) $x\in G\Rightarrow\frac{1}{x}=\frac{a-b\sqrt{3}}{a^2-3b^2}=a+(-b)\sqrt{3}\in G$.