

Soluție

1. a. $f(0) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, f(1) = \begin{pmatrix} 1 & 1 & 4 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{pmatrix}; f(0) + f(1) = \begin{pmatrix} 2 & 1 & 4 \\ 0 & 2 & 4 \\ 0 & 0 & 2 \end{pmatrix}.$

b. $f(1) \cdot f(-1) = f(1-1) = f(0) = I_3.$

c. $f(x+y) = \begin{pmatrix} 1 & x+y & 2(x+y)^2 + 2(x+y) \\ 0 & 1 & 4(x+y) \\ 0 & 0 & 1 \end{pmatrix}; f(x)f(y) = \begin{pmatrix} 1 & x+y & 2(x+y)^2 + 2(x+y) \\ 0 & 1 & 4(x+y) \\ 0 & 0 & 1 \end{pmatrix}.$

2.a. Adăugând la ambele părți ale ecuației $\hat{2} \cdot \hat{x} + \hat{1} \Leftrightarrow \hat{2} \cdot \hat{x} + \hat{5} + \hat{1} = \hat{1} + \hat{1} \Leftrightarrow \hat{2} \cdot \hat{x} = \hat{2} \Rightarrow \hat{x}_1 = \hat{1}$ și $\hat{x}_2 = \hat{4}.$

b. $\Delta = \begin{vmatrix} \hat{1} & \hat{2} & \hat{3} \\ \hat{2} & \hat{3} & \hat{1} \\ \hat{3} & \hat{1} & \hat{2} \end{vmatrix} = \hat{1} \cdot \hat{2} \cdot \hat{3} + \hat{2} \cdot \hat{1} \cdot \hat{3} + \hat{3} \cdot \hat{2} \cdot \hat{1} - \hat{3} \cdot \hat{3} \cdot \hat{3} - \hat{1} \cdot \hat{1} \cdot \hat{1} - \hat{2} \cdot \hat{2} \cdot \hat{2} = \hat{0} + \hat{0} + \hat{0} + \hat{3} + \hat{5} + \hat{4} = \hat{0}.$

c. $\hat{3}x = \hat{3}$ de unde $\hat{x} = \hat{1}, \hat{y} = \hat{2}$ sau $\hat{x} = \hat{3}, \hat{y} = \hat{4} \Rightarrow$ soluțiile $\hat{x} = \hat{1}, \hat{y} = \hat{2}$ sau $\hat{x} = \hat{3}, \hat{y} = \hat{4}$ sau $\hat{x} = \hat{5}, \hat{y} = \hat{0}.$