Soluție

1.a. $A=\begin{pmatrix}1&1&0\\0&1&1\\0&0&1\end{pmatrix}=\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}+\begin{pmatrix}0&1&0\\0&0&1\\0&0&0\end{pmatrix}=I_3+B$.

b. det A=$\begin{vmatrix}1&1&0\\0&1&1\\0&0&1\end{vmatrix}=1\Rightarrow A$ este inversabilă şi $A^{-1}=A^{*}=\begin{pmatrix}1&-1&1\\0&1&-1\\0&0&1\end{pmatrix}$.

c. Avem $\det X(a)=(a+1)^3$ şi relația din enunț devine $\det X(a)=(2a-1)^3\Rightarrow(a+1)^3=(2a-1)^3$
$\Leftrightarrow a+1=2a-1\Leftrightarrow a=2.$

2.a. $x*y=xy-x-y+1+1=x(y-1)-(y-1)+1=(x-1)(y-1)+1.$

b. $(x*y)*z=(xy-x-y+2)*z=(xy-x-y+2)\cdot z-(xy-x-y+2)-z+2=$
$=xyz-xy-yz-xz+x+y+z=x*(y*z).$

c. $\frac{\sqrt{1}}{2}*\frac{\sqrt{2}}{2}*\frac{\sqrt{3}}{2}*\frac{\sqrt{4}}{2}*\ldots*\frac{\sqrt{2009}}{2}=1,$ deoarece $a*1*b=1,\ \forall a,b\in\mathbb{R}.$