

Rezolvare

1.a. $A^2 = \begin{pmatrix} 2 & 0 & 1 \\ 1 & 1 & 1 \\ 1 & -1 & 2 \end{pmatrix} \begin{pmatrix} 2 & 0 & 1 \\ 1 & 1 & 1 \\ 1 & -1 & 2 \end{pmatrix} = \begin{pmatrix} 5 & -1 & 4 \\ 4 & 0 & 4 \\ 3 & -3 & 4 \end{pmatrix}.$

b. $\det(A) = \begin{vmatrix} 2 & a & 1 \\ 1 & 1 & 1 \\ 1 & -1 & 2 \end{vmatrix} = -a + 4.$ A inversabilă $\Leftrightarrow a \in \mathbb{R} \setminus \{4\}.$

c. Din pct. **b)**, pentru $a \in \mathbb{R} \setminus \{4\}$ avem $\det(A) \neq 0 \Rightarrow$ sistem Cramer $\Rightarrow x = 0 = y = z = 0$, soluție unică.

2.a. $x * y = y * x, \forall x, y \in \mathbb{Z} \Leftrightarrow p = 1.$

b. $x_1 = 0, x_2 = 4.$

c. $3x + 3y + 6 + q = 3x + q + 3y + q - 2 \Rightarrow q = 8.$