Rezolvare

1.a. Din $A^2=\begin{pmatrix}1&0&0\\0&3&0\\0&0&5\end{pmatrix}\cdot\begin{pmatrix}1&0&0\\0&3&0\\0&0&5\end{pmatrix}=\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}=I_3$.

b. $A\cdot X=I_3\Leftrightarrow\begin{pmatrix}1&0&0\\0&3&0\\0&0&5\end{pmatrix}\cdot\begin{pmatrix}a&b&c\\d&e&f\\g&h&l\end{pmatrix}=\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}\Leftrightarrow\begin{pmatrix}a&b&c\\3d&3e&3f\\5g&5h&5l\end{pmatrix}=\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}\Leftrightarrow X=\begin{pmatrix}1&0&0\\0&3&0\\0&0&5\end{pmatrix}$.

c. Avem $(B-A)^2=\begin{pmatrix}0&0&0\\2&0&0\\3&7&0\end{pmatrix}\cdot\begin{pmatrix}0&0&0\\2&0&0\\3&7&0\end{pmatrix}=\begin{pmatrix}0&0&0\\0&0&0\\6&0&0\end{pmatrix}$.

2.a. $x*e=e*x=x\Leftrightarrow 3xe+7x+7e+14=x\Leftrightarrow e(3x+7)=-6x-14\Leftrightarrow e=-2$ este element neutru.

b. $x*x\le -1\Leftrightarrow 3x^2+14x+15\le 0;\ \Delta=16,\ x_1=-\frac{5}{3},\ x_2=-3,\ x\in\mathbb{Z}\Rightarrow x\in\{-3,-2\}$.

c. $(x*y)*z=9xyz+21(xy+xz+yz)+49(x+y+z)+112=x*(y*z)$.