

Soluție

1. a. $A^2 = \begin{pmatrix} 1 & 2 & -3 \\ 1 & 2 & -3 \\ 1 & 2 & -3 \end{pmatrix} \cdot \begin{pmatrix} 1 & 2 & -3 \\ 1 & 2 & -3 \\ 1 & 2 & -3 \end{pmatrix} = O_3.$

b. $B^2 = \begin{pmatrix} 2a+1 & 4a & -6a \\ 2a & 4a+1 & -6a \\ 2a & 4a & -6a+1 \end{pmatrix} \Rightarrow 2B - B^2 = I_3.$

c. Din $2B - B^2 = I_3 \Rightarrow B(2I_3 - B) = I_3 \Rightarrow B^{-1} = 2I_3 - B.$

2.a. $x \circ y = 3xy + 3x + 3y + 3 + 2 - 3 = 3x(y+1) + 3(y+1) - 1 = (y+1)(3x+3) - 1 = 3(x+1)(y+1) - 1.$

b. $(x^2 - 5) \circ 6 = -1 \Leftrightarrow 3(x^2 - 4) \cdot 7 = 0 \Rightarrow x \in \{-2; 2\}.$

c. $a \circ b = 3(a+1)(b+1) - 1.$ Dacă $a+1 = \frac{2}{3}, b+1 = \frac{3}{2} \Rightarrow a \circ b = 2 \in \mathbb{N}.$