

Rezolvare

1.a. Din $A \cdot B = \begin{pmatrix} 4 & -2 & -2 \\ -2 & 4 & -2 \\ -2 & -2 & 4 \end{pmatrix} \cdot \begin{pmatrix} -2 & -2 & -2 \\ -2 & -2 & -2 \\ -2 & -2 & -2 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} = O_3.$

b. $\det(A) \cdot \det(B) = 0.$

c. $A^2 = \begin{pmatrix} 4 & -2 & -2 \\ -2 & 4 & -2 \\ -2 & -2 & 4 \end{pmatrix} \cdot \begin{pmatrix} 4 & -2 & -2 \\ -2 & 4 & -2 \\ -2 & -2 & 4 \end{pmatrix} = \begin{pmatrix} 24 & -12 & -12 \\ -12 & 24 & -12 \\ -12 & -12 & 24 \end{pmatrix} = 6A.$

$$B^2 = \begin{pmatrix} -2 & -2 & -2 \\ -2 & -2 & -2 \\ -2 & -2 & -2 \end{pmatrix} \cdot \begin{pmatrix} -2 & -2 & -2 \\ -2 & -2 & -2 \\ -2 & -2 & -2 \end{pmatrix} = \begin{pmatrix} 12 & 12 & 12 \\ 12 & 12 & 12 \\ 12 & 12 & 12 \end{pmatrix} = -6B.$$

2. a. $x \circ y = xy + 2x + 2y + 2 = xy + 2x + 2y + 4 - 2 = x(y + 2) + 2(y + 2) - 2 = (x + 2)(y + 2) - 2.$

b. $x \circ e = e \circ x = x \Leftrightarrow xe + 2x + 2e + 2 = x \Leftrightarrow e(x + 2) = -x - 2 \Leftrightarrow e = -1; -3 \circ x' = x' \circ (-3) = -1 \Leftrightarrow x' = -3.$

c. Sistemul se scrie $\begin{cases} x^2 + y^2 + 2 = 7 \\ (x^2 + 2)(y^2 + 2) - 2 = 16 \end{cases} \Leftrightarrow \begin{cases} x^2 + y^2 = 5 \\ (x^2 + 2)(y^2 + 2) = 18 \end{cases} \Leftrightarrow \begin{cases} x = 1 \\ y = 2 \end{cases} \text{ sau } \begin{cases} x = 2 \\ y = 1 \end{cases}.$