Soluție

1. a. Determinantul $\begin{vmatrix} \sqrt{2009}-1 & -1 \\ 1 & \sqrt{2009}+1 \end{vmatrix} = \left(\sqrt{2009}-1\right)\left(\sqrt{2009}+1\right)+1 = 2009$.

b. $\Delta = \begin{vmatrix} x_1 & x_2 \\ -x_2 & x_1 \end{vmatrix} = x_1^2 + x_2^2 = \left(x_1 + x_2\right)^2 - 2x_1x_2 = 4^2 - 2\cdot 2 = 12,$ unde am ținut cont de relațiile lui Viète.

c. $A^2 = \begin{pmatrix} -1 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} -1 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & -1 & 0 \\ 1 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, A^3 = A^2 \cdot A = \begin{pmatrix} 0 & -1 & 0 \\ 1 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} -1 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \Rightarrow$

$\Rightarrow A^3 + A^2 + A = 0_2$.

2. a. $x \circ y = 2xy - 8x - 8y + 36 = 2xy - 8x - 8y + 32 + 4 = 2x\left(y-4\right) - 8\left(y-4\right) + 4 = 2\left(x-4\right)\left(y-4\right) + 4.$

b. Ecuația se mai scrie $2\left(x-4\right)\left(x-4\right)+4 = 36 \Leftrightarrow \left(x-4\right)^2 = 16 \Leftrightarrow x_1 = 0,\ x_2 = 8.$

c. $\sqrt{1}\circ\sqrt{2}\circ\sqrt{3}\circ\ldots\circ\sqrt{2008} = \left(\sqrt{1}\circ\sqrt{2}\circ\sqrt{3}\circ\ldots\circ\sqrt{15}\right)\circ\sqrt{16}\circ\left(\sqrt{17}\circ\ldots\circ\sqrt{2009}\right) = a\circ 4\circ b = 4.$