Soluție

1. a. $A\cdot B=\begin{pmatrix}2&2\\0&2\end{pmatrix}\cdot\begin{pmatrix}x&y\\0&6\end{pmatrix}=\begin{pmatrix}2x&2y+12\\0&12\end{pmatrix}$ şi $B\cdot A=\begin{pmatrix}x&y\\0&6\end{pmatrix}\cdot\begin{pmatrix}2&2\\0&2\end{pmatrix}=\begin{pmatrix}2x&2x+2y\\0&12\end{pmatrix}$.

$A\cdot B=B\cdot A\Rightarrow x=6$ şi $y\in\mathbb{R}$.

b. Calculând $A^2=\begin{pmatrix}4&8\\0&4\end{pmatrix}$ şi $4(A-I_2)=4\begin{pmatrix}1&2\\0&1\end{pmatrix}=\begin{pmatrix}4&8\\0&4\end{pmatrix}\Rightarrow A^2=4(A-I_2)$.

c. $A=2\begin{pmatrix}1&1\\0&1\end{pmatrix}$, $A^2=2^2\begin{pmatrix}1&1\\0&1\end{pmatrix}\cdot\begin{pmatrix}1&1\\0&1\end{pmatrix}=2^2\begin{pmatrix}1&2\\0&1\end{pmatrix}$, $A^3=2^3\begin{pmatrix}1&2\\0&1\end{pmatrix}\cdot\begin{pmatrix}1&1\\0&1\end{pmatrix}=2^3\begin{pmatrix}1&3\\0&1\end{pmatrix}$.

Din relația $A^3-aA^2+4A=O_2\Rightarrow a=4$.

2. a. $x*y=xy-3(x+y)+12=xy-3x-3y+9+3=x(y-3)-3(y-3)+3=(x-3)(y-3)+3,\forall x,y\in\mathbb{R}$.

b. Ecuația $x\circ(x+1)+x*(x+1)=11\Leftrightarrow 2x+4+x^2-5x+6+3=11\Leftrightarrow$

$\Leftrightarrow x^2-3x+2=0$ cu soluțiile $x_1=1$ şi $x_2=2$.

c. $\begin{cases}x\circ(y-1)=0\\(x+1)*y=x*(y+1)\end{cases}\Leftrightarrow\begin{cases}x+y+2=0\\(x-2)(y-3)=(x-3)(y-2)\end{cases}\Leftrightarrow$

$\Leftrightarrow\begin{cases}x+y+2=0\\xy-3x-2y+6=xy-2x-3y+6\end{cases}\Leftrightarrow\begin{cases}x+y+2=0\\x=y\end{cases}\Leftrightarrow x=y=-1.$