

Soluție

1.a. $A^2 = \begin{pmatrix} 4 & 8 \\ 2 & 4 \end{pmatrix} \cdot \begin{pmatrix} 4 & 8 \\ 2 & 4 \end{pmatrix} = \begin{pmatrix} 32 & 64 \\ 16 & 32 \end{pmatrix} = 8 \begin{pmatrix} 4 & 8 \\ 2 & 4 \end{pmatrix} = 8A.$

b. Avem $\det X(a) = \begin{vmatrix} 4a+1 & 8a \\ 2a & 4a+1 \end{vmatrix} = (4a+1)^2 - 16a^2 = 16a^2 + 8a + 1 - 16a^2 = 8a + 1.$

c. $X(a) \cdot X(b) = \begin{pmatrix} (4a+1)(4b+1) + 8a \cdot 2a & 32ab + 8b + 32ab + 8a \\ 2a(4b+1) + 2b(4a+1) & 2a \cdot 8b + (4a+1)(4b+1) \end{pmatrix} =$
 $= \begin{pmatrix} 4(8ab + a + b) + 1 & 8(8ab + a + b) \\ 2(8ab + a + b) & 4(8ab + a + b) + 1 \end{pmatrix} = X(a + b + 8ab).$

2.a. Din $f(1) = 3^{670} - 1$ și $f(-1) = (-1)^{670} - 1 = 0 \Rightarrow f(1) + f(-1) = 3^{670} - 1.$

b. $f(1) = a_0 + a_1 + a_2 + \dots + a_{2009} = 3^{670} - 1 = \text{număr par}.$

c. $r(X) = \frac{3^{670} - 1}{2} \cdot X + \frac{3^{670} - 1}{2}.$