

Rezolvare

1.a. $\lim_{x \rightarrow 1} f(x) = \frac{1 + \ln 1}{1 - \ln 1} = 1.$

b. $f'(x) = \frac{2}{x(1 - \ln x)^2}.$

c. $\lim_{x \rightarrow \infty} f(x) = -1 \Rightarrow y = -1$ ecuația asimptotei orizontale.

2.a. $\int (f + g)(x) dx = e^x + \ln x + C.$

b. $\int_1^2 (f^2(x) + g^2(x)) dx = \left(\frac{e^{2x}}{2} - \frac{1}{x} \right) \Big|_1^2 = \frac{e^4 - e^2 + 1}{2}.$

c. Din $f(x)g(x) \leq \frac{f^2(x) + g^2(x)}{2} \Rightarrow \int_1^2 f(x)g(x) dx \leq \frac{1}{2} \int_1^2 (f^2(x) + g^2(x)) dx.$ Deci $\int_1^2 e^x \frac{1}{x} dx \leq \frac{e^4 - e^2 + 1}{4}.$