

Soluție

1.a) $f'(x) = \frac{-2}{(x-1)^2}.$

b) $\lim_{x \rightarrow -1} \frac{f(x) - f(-1)}{x + 1} = f'(-1) ; f'(-1) = \frac{-2}{4} = -\frac{1}{2}.$

c) $\lim_{x \rightarrow +\infty} f(x) = 1 \Rightarrow y = 1$ asimptotă orizontală spre $+\infty$.

2.a) $\int_0^1 e^{-x} \cdot x \cdot e^x dx = \int_0^1 x dx = \frac{1}{2}.$

b) $I_1 = \int_0^1 x e^x dx = \left(x e^x - e^x \right) \Big|_0^1 = 1.$

c) $I_n = \int_0^1 x^n e^x dx = x^n e^x \Big|_0^1 - n \int_0^1 x^{n-1} e^x dx = e - n I_{n-1} \Rightarrow I_n + n I_{n-1} = e.$