Soluție

1.a) $f'(x)=\dfrac{-4}{(x-3)^2}$.

b) $\lim\limits_{x\to 4}\dfrac{f(x)-f(4)}{x-4}=f'(4)=-4$.

c) $\lim\limits_{x\to\infty}f(x)=1\Rightarrow y=1$ asimptotă orizontală.

2.a) $\int\limits_0^1 f(x)\,dx=\ln(x+1)\Big|_0^1=\ln 2$.

b) $V=\pi\int\limits_0^2\dfrac{1}{(x+1)^2}\,dx=\pi\cdot\dfrac{-1}{x+1}\Big|_0^2=\pi\left(\dfrac{-1}{3}+1\right)=\dfrac{2\pi}{3}$.

c) $a\le x\le a+1\Leftrightarrow a+1\le x+1\le a+2\Leftrightarrow\dfrac{1}{a+2}\le\dfrac{1}{x+1}\le\dfrac{1}{a+1}\Rightarrow\int\limits_a^{a+1}\dfrac{1}{a+2}\,dx\le\int\limits_a^{a+1}f(x)\,dx\le\int\limits_a^{a+1}\dfrac{1}{a+1}\,dx$

$\Leftrightarrow\dfrac{1}{a+2}\cdot x\Big|_a^{a+1}\le\int\limits_a^{a+1}f(x)\,dx\le\dfrac{1}{a+1}\cdot x\Big|_a^{a+1}$.