Soluție

1.a) $h(x)=\dfrac{(x-2)+(x-1)}{(x-1)(x-2)}=\dfrac{1}{x-1}+\dfrac{1}{x-2}$.

b) $h'(x)=\dfrac{-1}{(x-1)^2}+\dfrac{-1}{(x-2)^2}<0\Rightarrow$ h este descrescătoare pe fiecare din intervalele $(-\infty;1),(1;2),(2;\infty)$.

c) $h'(x)<0,\ \forall x\in\mathbb{R}\setminus\{1;2\}$ și $h(x)=\dfrac{f'(x)}{f(x)}\Rightarrow\left(\dfrac{f'(x)}{f(x)}\right)'<0\Rightarrow\left(f'(x)\right)^2\geq f(x)\cdot f''(x)$.

2.a) $V=\pi\int_1^3 x^2dx=\pi\left.\dfrac{x^3}{3}\right|_1^3=\pi\left(9-\dfrac{1}{3}\right)=\pi\cdot\dfrac{26}{3}$.

b) $\int f(x)dx=\dfrac{x^{2010}}{2010}+\dfrac{x^2}{2}+x+\mathcal{C};\ F(0)=1\Leftrightarrow F(x)=\dfrac{x^{2010}}{2010}+\dfrac{x^2}{2}+x+1$.

c) $\int_0^x f(t)\,dt=\dfrac{x^{2010}}{2010}+\dfrac{x^2}{2}+x\ \Rightarrow\lim\limits_{x\to\infty}\dfrac{\int_0^x f(t)dt}{x^{2010}}=\dfrac{1}{2010}$.