Soluție

1.a) f continuă $\Leftrightarrow l_s(0)=l_d(0)=f(0)$; $l_s(0)=1=f(0)$, $l_d(0)=1 \Rightarrow f$ este continuă în $x=0$.

b) $x\in(-\infty,0]$; $f'(x)=\dfrac{-2x}{\left(x^2+1\right)^2}$; $f'(x)\geq 0$ oricare ar fi $x\in(-\infty,0] \Rightarrow f$ crescătoare pe $(-\infty\,,\,0]$.

c) $f(-1)=\dfrac{1}{2} \Rightarrow A\in G_f$; ecuația tangentei: $y-f(-1)=f'(-1)(x+1)$; $f'(-1)=\dfrac{1}{2} \Rightarrow y=\dfrac{1}{2}x+1$.

2.a) $f_1\left(\sqrt{x-1}\right)=\dfrac{1}{x}\Rightarrow\displaystyle\int_1^e\frac{1}{x}dx=\ln x\Big|_1^e=\ln e-\ln 1=1$.

b) $f_2(x)=\dfrac{1}{\left(x^2+1\right)^2}\Rightarrow g(x)=\left(x^2+1\right)^2=x^4+2x^2+1$; $\displaystyle\int g(x)\,dx=\frac{x^5}{5}+\frac{2x^3}{3}+x+\mathcal{C}$. O primitivă este

$G(x)=\dfrac{x^5}{5}+\dfrac{2x^3}{3}+x+k$; $G(1)=\dfrac{13}{15}\Leftrightarrow\dfrac{1}{5}+\dfrac{2}{3}+1+k=\dfrac{13}{15}\Leftrightarrow k=-1\Rightarrow G(x)=\dfrac{x^5}{5}+\dfrac{2x^3}{3}+x-1$.

c) $\displaystyle\int_0^1 x\,f_n(x)\,dx=\int_0^1\frac{x}{\left(x^2+1\right)^n}\,dx=\int_1^2\frac{1}{t^n}\cdot\frac{1}{2}\,dt=\frac{1}{2}\cdot\frac{t^{-n+1}}{-n+1}\Bigg|_1^2=\frac{1}{2(1-n)}\left(\frac{1}{2^{n-1}}-1\right)$.