Soluții

1. a) $f'(x)=3^x\ln 3-\left(\frac{1}{2}\right)^x\ln\frac{1}{2}$.

b) $\lim\limits_{x\to 0}\frac{f(x)-f(0)}{x}=f'(0)=\ln 6$.

c) $f'(x)=3^x\ln 3+\left(\frac{1}{2}\right)^x\ln 2>0$ pentru orice $x\in\mathbb{R}$, deci f este crescătoare pe $\mathbb{R}$.

2. a) $\int f(x)dx=\frac{x^2}{2}+\ln x+\mathcal{C}$.

b) $V=\pi\int\limits_1^2\left(x+\frac{1}{x}\right)^2dx=\pi\left(\frac{x^3}{3}+2x-\frac{1}{x}\right)\Big|_1^2=\frac{29\pi}{6}$.

c) $\int\limits_1^e f(x)\ln x dx=\int\limits_1^e x\ln x dx+\int\limits_1^e\frac{1}{x}\ln x dx=\left(\frac{x^2}{2}\ln x-\frac{x^2}{4}\right)\Big|_1^e+\frac{\ln^2 x}{2}\Big|_1^e=\frac{e^2+3}{4}$.