Soluții

1. a) $f'(x)=\dfrac{2x^2-x^2-1}{x^2}=\dfrac{x^2-1}{x^2}$.

b) $\lim\limits_{x\to+\infty}\dfrac{f(x)}{x}=1$, $\lim\limits_{x\to+\infty}\left(f(x)-x\right)=0$ deci $y=x$ este asimptotă oblică la $+\infty$.

c) $f''(x)=\dfrac{2}{x^3}>0$, pentru orice $x>0$, deci f este convexă pe $(0,+\infty)$.

2. a) $\int f_0(x)\cdot \mathrm{e}^{-x}\,dx=\dfrac{x^2}{2}+x+\mathcal{C}$.

b) $f_1(x)=\left(x^2+1\right)e^x\ge 0,\ \forall x\in[0;1]\Rightarrow \text{Aria}\left(\Gamma_{f_1}\right)=\int\limits_0^1\left(x^2+1\right)\cdot e^x dx=\left(x^2-2x+3\right)\cdot e^x\Big|_0^1=2e-3$.

c) $\int\limits_0^1 f_{2008}(x)dx+\int\limits_0^1 f_{2010}(x)dx=\int\limits_0^1\left(x^{2009}\left(1+x^2\right)+2\right)\cdot e^x dx\ge\int\limits_0^1\left(x^{2009}\cdot 2x+2\right)\cdot e^x dx=2\int\limits_0^1 f_{2009}(x)dx$.