Soluții

1.a) f este continuă pe D, deci în aceste puncte nu avem asimptote verticale
$f_d(-2)=-\infty,\ f_s(2)=\infty \Rightarrow x=-2, x=2$ sunt asimptote verticale.

b) $f''(x)=\dfrac{8x}{\left(4-x^2\right)^2}$,deci $x=0$ este punct de inflexiune.

c) Limita cerută este $L=\lim\limits_{x\to\infty} x^a\cdot\ln\left(\dfrac{2+\frac{1}{x}}{2-\frac{1}{x}}\right)=\lim\limits_{y\searrow 0}\dfrac{1}{y^a}\cdot\ln\dfrac{2+y}{2-y}$.

$$L=\lim_{y\searrow 0}\frac{1}{y^a}\cdot\frac{\ln\left(1+\frac{2y}{2-y}\right)}{\frac{2y}{2-y}}\cdot\frac{2y}{2-y}=\lim_{y\searrow 0}\frac{1}{y^{a-1}}=\begin{cases}0, a<1\\ 1, a=1\\ \infty, a>1\end{cases}.$$

2.a) $I=\int_0^1\left(-x+2-\dfrac{x}{x^2+4}\right)dx=\left(-\dfrac{x^2}{2}+2x-\dfrac{1}{2}\cdot\ln\left(x^2+4\right)\right)\Big|_0^1=\dfrac{3}{2}-\dfrac{1}{2}\ln\dfrac{5}{4}$.

b) Avem că $I=\int_1^4\dfrac{x^2}{\left(x^2+4\right)^2}dx=\dfrac{-1}{2}\cdot\int_1^4\left(\dfrac{1}{x^2+4}\right)'\cdot x dx$. $I=\dfrac{1}{2}\cdot\dfrac{-x}{x^2+4}\Big|_1^4+\dfrac{1}{4}\cdot\text{arctg}\dfrac{x}{2}\Big|_1^4=\dfrac{1}{4}\left(\text{arctg}2-\text{arctg}\dfrac{1}{2}\right)$.

c) Cu substituția $f^{-1}(x)=t,\ I=\int_{\frac{4}{5}}^{2} f^{-1}(x)dx=-\int_0^1 t\cdot f'(t)\,dt=-t\cdot f(t)\Big|_0^1+\int_0^1 f(t)dt=\dfrac{7}{10}-\dfrac{1}{2}\ln\dfrac{5}{4}$.