**Soluții**

**1. a)** $f$ este derivabilă pe $[0,\infty)$, $f'(x)=2e^x+6x-2$. $f'(x)\geq 0, x\geq 0$, cu egalitate dacă $x=0$, de unde se obține concluzia.

**b)** Pentru $x<0,\ f'(x)<0$, deci 0 este punct de minim global, de unde $f(x)\geq f(0),\ \forall x\in\mathbb{R}$.

**c)** Deoarece $f'(x)=2e^x+6x-2$, avem $\lim\limits_{x\to\infty}\frac{f'(x)}{f(x)}=\lim\limits_{x\to\infty}\frac{2e^x+6x-2}{2e^x+3x^2-2x+5}=1$.

**2. a)** $I=\int_0^1\frac{1}{1+t^2}dt=\operatorname{arctg}t\Big|_0^1=\frac{\pi}{4}$.

**b)** Cu substitutia $\frac{1}{t}=y\Rightarrow J=\int_{\frac{1}{x}}^1 f(t)dt=\int_x^1 f\left(\frac{1}{y}\right)\cdot\left(\frac{-1}{y^2}\right)dy=\int_1^x\frac{1}{t^2}\cdot f\left(\frac{1}{t}\right)dt=\int_1^x t^3 f(t)\,dt$.

**c)** $A=\int_{\frac{1}{x}}^x f(t)dt=\int_{\frac{1}{x}}^1 f(t)dt+\int_1^x f(t)dt=\int_1^x t^3 f(t)dt+\int_1^x f(t)dt=\int_1^x\left(t^3+1\right)f(t)\,dt$.

$A=\int_1^x\frac{1}{1+t^2}dt=\operatorname{arctg}x-\operatorname{arctg}1$, deci limita ceruta este $\frac{\pi}{4}$.