Rezolvare

1. **a**) $f'(x)=\dfrac{1}{(x+2)^2+1}-\dfrac{1}{x^2+1}$.

b) $x+2>x\Rightarrow \operatorname{arctg}(x+2)>\operatorname{arctg}x\Rightarrow f(x)>0$.

f este strict crescătoare pe intervalul $(-\infty,-1)$ şi strict descrescător are pe intervalul $[-1,\infty)$ deci -1 este maxim global. $f(-1)=\operatorname{arctg}1-\operatorname{arctg}(-1)=\dfrac{\pi}{4}+\dfrac{\pi}{4}=\dfrac{\pi}{2}$.

c) Se arată că $g'(x)=0,\ \forall x\in\mathbb{R}$.

2. **a**) $\displaystyle\int_1^2\frac{x^2-1+\frac{1}{1+x^2}}{x}dx=\frac{x^2}{2}\Big|_1^2-\ln x\Big|_1^2-\frac{1}{2}\ln\left(x^2+1\right)\Big|_1^2=\frac{3}{2}-\frac{1}{2}\ln 5+\frac{1}{2}\ln 2$.

b) $f(x)\geq\dfrac{x^3}{3}-x-\dfrac{\pi}{2}\rightarrow\displaystyle\int_0^x f(t)dt\geq\frac{x^4}{12}-\frac{x^2}{2}-\frac{\pi}{2}\cdot x$. Deci $\displaystyle\lim_{x\to\infty}\frac{\int_0^x f(t)dt}{x^3}=+\infty$.

c) $g(x)\geq f(x),\ \forall x\in[0,1)\Rightarrow \mathrm{A}=\displaystyle\int_0^1\left(g(x)-f(x)\right)dx=\left(\frac{x^2}{2}-\frac{x^4}{12}\right)\Big|_0^1=\frac{1}{2}-\frac{1}{12}=\frac{5}{12}$.