rezolvare

1.a) $f_d'(1)=\lim\limits_{x\searrow 1} f'(x)=\lim\limits_{x\searrow 1}\frac{2-x}{e^x}=\frac{1}{e}$ și $f_s'(1)=\lim\limits_{x\nearrow 1} f'(x)=\lim\limits_{x\nearrow 1}\frac{x-2}{e^x}=-\frac{1}{e}$.

b)

x	$-\infty$		1		2		∞
$f'(x)$	$-$	$-$	|	$+$	0	$-$	$-$
$f(x)$	∞	$\searrow$	0	$\nearrow$	$1/e^2$	$\searrow$	0

Pentru $m<0$ nu avem soluții, pentru $m=0$ sau $m>1/e^2$ avem o soluție, pentru $m=1/e^2$ avem două soluții, iar pentru $0<m<1/e^2$ avem trei soluții.

c) $\frac{1}{e^2}+\frac{2}{e^3}+...+\frac{n-1}{e^n}=\left(\frac{1}{e^2}+\frac{1}{e^3}+...+\frac{1}{e^n}\right)+\left(\frac{1}{e^3}+\frac{1}{e^4}+...+\frac{1}{e^n}\right)+...+\left(\frac{1}{e^{n-1}}+\frac{1}{e^n}\right)+\frac{1}{e^n}=$

$$=\frac{1}{e^2}\frac{1-\frac{1}{e^{n-1}}}{1-\frac{1}{e}}+\frac{1}{e^3}\frac{1-\frac{1}{e^{n-2}}}{1-\frac{1}{e}}+...+\frac{1}{e^{n-1}}\frac{1-\frac{1}{e^2}}{1-\frac{1}{e}}+\frac{1}{e^n}\frac{1-\frac{1}{e}}{1-\frac{1}{e}}=\frac{1}{1-\frac{1}{e}}\left(\frac{1}{e^2}+\frac{1}{e^3}+...+\frac{1}{e^n}\right)-\frac{n}{1-\frac{1}{e}}\frac{1}{e^n}\to\frac{1}{(e-1)^2}.$$

2.a) $F'(x)=(2a+c)x\cos x-ax^2\sin x+(c-b)\sin x\Rightarrow a=-1, b=c=2$.

b) $\int_{\frac{1}{\pi}}^{\frac{2}{\pi}}\frac{1}{4x^2}\sin\frac{1}{2x}=\frac{1}{2}\cos\frac{1}{2x}\Big|_{\frac{1}{\pi}}^{\frac{2}{\pi}}=\frac{\sqrt{2}}{4}$.

c) Observăm că $f(x)<g(x)$, $\forall x\in\left(0,\frac{\pi}{2}\right)$, adică $x\sin x+x<\pi$, $\forall x\in\left(0,\frac{\pi}{2}\right)$. Avem $A=\int_0^{\frac{\pi}{2}}\left(g(x)-f(x)\right)dx=$

$$=\left(\frac{\pi x^2}{2}-\frac{x^3}{3}+x^2\cos x-2x\sin x-2\cos x\right)\Big|_0^{\frac{\pi}{2}}=\frac{\pi^3}{12}-\pi+2.$$