Soluție

1.a) $\lim\limits_{x\to\infty} f(x) = \frac{\pi}{2}$ deci $y = \frac{\pi}{2}$ este asimptotă orizontală spre $+\infty$.

b) $g'(x) = f'(x+1) - f'(x) - f'\left(\frac{1}{x^2+x+1}\right)\cdot\left(\frac{1}{x^2+x+1}\right)' = 0 \Rightarrow g =$ constană $= g(0) = 0$

c) $\operatorname{arctg}\frac{1}{k^2+k+1} = \operatorname{arctg}(k+1) - \operatorname{arctg}k$, deci $\sum\limits_{k=1}^{n}\operatorname{arctg}\frac{1}{k^2+k+1} = \operatorname{arctg}(n+1) - \frac{\pi}{4}$, iar

$\lim\limits_{n\to\infty}(\operatorname{arctg}(n+1) - \frac{\pi}{4}) = \frac{\pi}{4}$.

2.a) $I_1 = \int_0^1 e^{-x}xdx = \int_0^1 x(e^{-x})'dx = -xe^{-x}\Big|_0^1 + \int_0^1 e^{-x}dx = -e^{-1} - e^{-x}\Big|_0^1 = -2e^{-1} + 1$.

b) $I_n = \int_0^1 \left(-e^{-x}\right)' x^n dx = -x^n e^{-x}\Big|_0^1 + n\int_0^1 e^{-x}x^{n-1}dx = -e^{-1} + nI_{n-1}$.

c) Avem $0 \le I_n \le \int_0^1 x^n dx = \frac{1}{n+1}$, deci $\lim\limits_{n\to\infty} I_n = 0$.