Soluție

1. a) $f'(x)=\dfrac{3x^2+6x+2}{3\sqrt[3]{(x^3+3x^2+2x+1)^2}}-\dfrac{3x^2-1}{3\sqrt[3]{(x^3-x+1)^2}}$

$f(0)=0,\ \ f'(0)=1$, deci ecuația este $y=x$.

b) $\lim\limits_{x\to\infty}f(x)=\lim\limits_{x\to\infty}\dfrac{3x^2+3x}{\sqrt[3]{(x^3+3x^2+2x+1)^2}+\sqrt[3]{(x^3+3x^2+2x+1)\left(x^3-x+1\right)}+\sqrt[3]{(x^3-x+1)^3}}=1$, deci

$y=1$ este asimptota spre $+\infty$.

c) Din $f(k)=\sqrt[3]{k(k+1)(k+2)+1}-\sqrt[3]{(k-1)k(k+1)+1}$ deducem $\sum\limits_{k=1}^{n}f(k)=\sqrt[3]{n^3+3n^2+2n+1}-1$.

Limita cerută este de tipul 1^∞ și devine $l=e^{\lim\limits_{n\to\infty}(\sqrt[3]{n^3+3n^2+2n+1}-n-1)}=e^0=1$.

2.a) $f_1(e)=\int_{\frac{1}{e}}^{e}\left(\dfrac{t^2}{2}\right)'\ln t dt=\dfrac{t^2\ln t}{2}\Bigg|_{\frac{1}{e}}^{e}-\dfrac{1}{2}\int_{\frac{1}{e}}^{e}tdt=\dfrac{e^2}{4}+\dfrac{3}{4e^2}$.

b) $f_n'(x)=x^n\ln x$, iar $x\in(0;1)\Rightarrow \ln x\le 0\Rightarrow f'(x)\le 0$

c) $-1\le\ln t\le 0\ \forall \mathrm{x}\in\left[\dfrac{1}{\mathrm{e}};1\right]$, rezultă $-\int_{\frac{1}{\mathrm{e}}}^{1}t^n\le f_n(1)\le 0$, adică $-\dfrac{1}{n+1}+\dfrac{1}{(n+1)e^{n+1}}\le f_n(1)\le 0$, deci

$\lim\limits_{n\to\infty}f_n(1)=0$